

Analysis of students error in mathematical problem solving based on Newman's error analysis

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Abstract. After conducting literature studies, the researchers found that more studies on problem solving focused on the application of learning methods and the development of teaching materials to improve students problem-solving ability rather than identification and discovery of student errors. This study aims to describe students error according to Newman's Error Analysis in solving derivative of algebraic function problem. This research used a qualitative approach with case study design and purposive sampling technique. Data were collected through problem-solving test on derivative of algebraic function and interview of senior high school students grade 11 in Bandung. The results of this study showed that students did five types of errors in solving the problem of derivative of algebraic function that were comprehension error, transformation error, process skill error, encoding error and careless. Analysis of student error was expected to help to reflect on solving mathematical problems and became a reference for teachers in choosing strategy, model or learning media to reduce errors made by students.

1. Introduction

Problem solving is one of the five key abilities in mathematics learning that students should possess, as mentioned by the National Council of Teachers of Mathematics [1]. Problem solving is one of the main focuses of mathematics learning and the essential ability students must possess, as problem-solving capability represents high-level, complex mental activities comprised of various ability, cognitive and affective. Problem solving also encourages students to have high thinking ability such as visualization, association, abstraction, understanding, manipulation, reasoning, analysis, synthesis, and generalization which need to be exercise and coordinated [2,3,4,5]. Problem solving plays an important role in mathematics educations [6]. The above description confirms how important the problem-solving ability is in the students' learning of mathematics.

Despite the fact that, the ability to solve mathematical problems is important, the facts in the field show that the solution of mathematical problems of students has not been as expected. It was raised by the center of development and empowerment of mathematics educators and education personnel that Indonesian students are weak in solving problems that demanded mathematical problem-solving ability [7]. In addition, the results of several previous studies found that students' mathematical problem-solving ability is still low [8,9].

International studies on problem solving focus on problem-solving approaches and illustrate the success of mathematical problem solving [10]. Similarly in Indonesia, much of the research on

problem solving focuses on the application of learning methods and the development of teaching materials to improve students' problem-solving ability rather than the identification and discovery of the underlying causes of students' mathematical problem-solving abilities. For example, Windari's research to improve mathematical problem-solving abilities using learning strategy and research done by Apiati developed mathematics teaching materials based on realistic mathematics learning approach [8,11]. To improve mathematical problem-solving ability, researcher must know the root that cause the low mathematical problem-solving ability of a student by among others identifying and analyzing students' errors in solving problems that cause the problem-solving ability to be low [12].

Newman's Error Analysis can be used in identifying and analyzing student errors in solving mathematical problems. Newman's Error Analysis is a framework with a simple diagnostic procedure that includes reading, comprehension, transformation, ability and encoding processes. Newman classifies the errors students make as follows: reading error is when students make mistakes in reading important words in the question or students are wrong in reading the main information, so that students do not use the information to solve the problem. Comprehension error is the second type of error in which students have read the problem well, but do not understand the meaning of the question (e.g. cannot identify known and asked of the problem). Transformation error is a third type of error which is when students make mistakes in changing the problem into a mathematical model such as equations, drawings, graphic or table. Process skill error is a student's mistake in choosing rules/procedures or students already using correct procedures/rules, but errors occur in the calculation or computation. Encoding error is the fifth kind of mistake that students in this case make mistakes in writing the answer correctly, cannot show the truth of the answer or do not write the conclusion of the answer. Careless is the sixth mistake that is also called error due to carelessness or inadequate [13].

Identifying and analyzing student errors is needed as a reference to choose appropriate strategies, models, and instructional media to reduce and even eliminate student mistakes. This is the same as the study of Pramudya and colleagues in which they identified and analyzed students' errors in solving geometry problems using Newman's Error Analysis based on spatial intelligence [14]. Just like Pramudya et al, Rohman and Sutiarto analyzed problem-solving ability using Newman's Error Analysis on the topic of equation system with two variables [15]. By using Newman's Error Analysis, identifying and analyzing can be more directed and orderly [13]. Taking those into account, the researcher has done the research "Analysis of student error in mathematical problem-solving based on Newman's Error Analysis". The purpose of this study is to describe student error based on Error Analysis in solving derivative of algebraic function problems.

2. Method

This research used qualitative approach with case study design. The sampling technique used was purposive sampling. The test instrument in this research was three items to identify problem solving abilities in derivative of algebraic function. The types of problems used were two types of well-defined problems and one type of ill-define problem in the context of mathematics. Data analysis in this research used Interactive Analysis models by Miles, Huberman and Saldaña with some steps of analysis activity i.e data collection, data reduction, data presentation, and conclusions [16]. The data would be classified and identified into six types of errors according to Newman namely reading error, comprehension error, transformation error, process skills error, encoding error and careless. This research was conducted for 36 students of class 11 majoring in science in one of senior high schools in Bandung Indonesia..

3. Result and Discussion

The error in solving a mathematical problem is a deviation from the accuracy or rules in solving math problems. This study discusses students' errors in derived algebra material. Errors were analyzed based on students' errors in solving process and interview responds. The results showed that there were several types of mistakes made by students. The first error being identified was a reading error. Reading error is an error when students are wrong in reading important words in the question or

students are wrong in reading information [13]. However, in the aspect of reading error, there was no mistake made by the student.

The second error identified was a misunderstanding. Misunderstanding is a student's mistake when he/she can read the problem well but cannot understand the meaning of symbols or questions. It also includes inability to identify what is- known and asked of the problem [13]. In this study, 25.9% of students misunderstood. One of the students' misconceptions in this study is shown in Figure 1. Students were asked to determine the maximum volume that a tube had if it was placed in a sphere of radius r meter. Students made misconception because students did not identify what was known and asked of the problem, but students had tried to transform the problem into an image, but the image was wrong. The student assumed that the known r on the question was the radius of the tube, whereas r was the radius of the sphere. Such misunderstandings occurred because students had problems with understanding the meaning or purpose of the given problem. This was because they misunderstood certain keywords, or they had difficulty in using the correct information and less accustomed to writing what was known and asked in the question and the tendency of the students to shorten the writing of the answer [14,16,18].

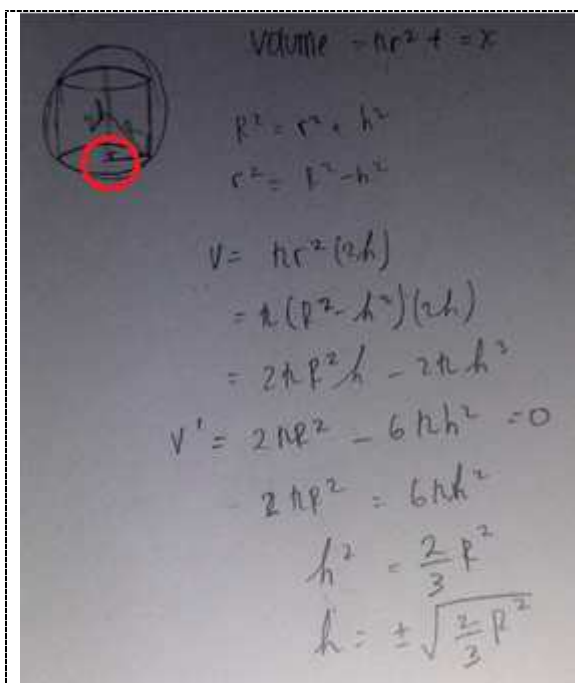


Figure 1. Example of Student's Comprehension Error

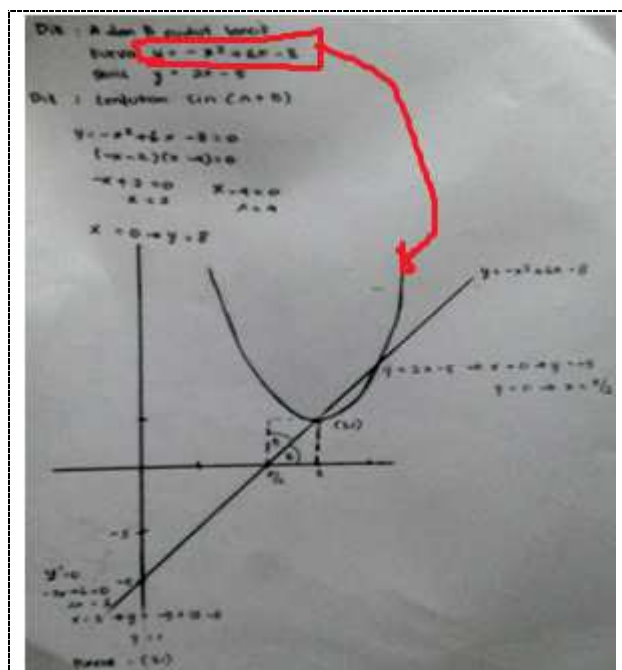


Figure 2. Example of Student's Transformation Error

The transformation error is an error when it is identified. The transformation error is an error when the student incorrectly transforms the problem into a mathematical model such as equation, image, graphic or table [13]. In this study, 22.4% of students made a transformation error. One of the transformational mistakes made by students in this study was shown in Figure 2. Students were asked to determine the number of sinus of two tapered angles formed by the tangent line of a curve with the x axis. Students made a transformation error because the student incorrectly drew the curve chart $y = -x^2 + 6x - 8$. The students drew the open curve upwards, while it should be open down because a value of the curve was negative. In addition, the students did not make a line of the curve with the line $y - 2x + 5 = 0$ and because of the error, students did not find two tapered angle that was formed. Transformation errors occurred because of students' low ability in manipulating data into a mathematical model, this was because they were not accustomed to do so [16,17].

The fourth error being identified is the process skill error. Process skill error is the student's mistake in choosing the rules/procedures or the students are already using the correct procedures/rules but

making mistakes in calculating or computing [13]. In this study, 26.2% of students perform process skill error. One of the process skill errors students performed in this study is shown in Figure 3. Students were asked to determine the normal line equation on the $f(x)$ function curve of $(f \circ g)(x) = -4x^2 - 12x - 5$ and $g(x) = 2x + 5$ through point (2,5). The students misconstrued the process skill error because the students incorrectly determined and used the procedure to determine the function of $f(x)$. Most of the process skill errors occurred because students incorrectly used the concepts in solving problems or the students were correct using the concepts to solve the problem, but not doing the right thing, or the students' lack of understanding of the prerequisite materials needed to solve the problem, or the lack of practice in solving mathematics problem [15,17,18].

Dik: $(f \circ g)(x) = -4x^2 - 12x - 5$
 $g(x) = 2x + 5$
 Dit: Tentukan persamaan garis normal pada Kurva $f(x)$ melalui titik (2,5)
 Jwb: $f(x) = -4(2x+5)^2 - 12(2x+5) - 5$
 $= -4(4x^2 + 20x + 25) - 24x - 60 - 5$
 $= -16x^2 - 20x + 25 - 24x - 60 - 5$
 $= -16x^2 - 44x - 40$
 $f'(x) = -32x - 44$
 $f'(2) = -32(2) - 44 = -64 - 44 = -108$
 $f(2) = -16(2)^2 - 44(2) - 40 = -64 - 88 - 40 = -192$
 $y - y_1 = m(x - x_1)$
 $y - (-192) = -108(x - 2)$
 $y + 192 = -108x + 216$
 $y = -108x + 216 - 192$
 $y = -108x + 24$

Figure 3. Example of Student's Process Skill Error

Diagram: Bola (sphere) and Silinder (cylinder) with radius R and height H.
 R : jari-jari bola
 r : jari-jari silinder
 H : tinggi silinder
 $\text{Volume} = \pi r^2 H$
 $r^2 = R^2 - \frac{1}{4}H^2$
 $V = \pi(R^2 - \frac{1}{4}H^2)H$
 $V = \pi R^2 H - \frac{1}{4}\pi H^3$
 $V'(H) = \pi R^2 - \frac{3}{4}\pi H^2 = 0$
 $\pi R^2 = \frac{3}{4}\pi H^2$
 $H = \sqrt{\frac{4}{3}R^2}$
 $H = \frac{2R}{\sqrt{3}}$
 $V_{maks} = \pi R^2 H - \frac{1}{4}\pi H^3$
 $= \pi R^2 \left(\frac{2R}{\sqrt{3}}\right) - \frac{1}{4}\pi \left(\frac{2R}{\sqrt{3}}\right)^3$
 $= \frac{2\pi R^3}{\sqrt{3}} - \frac{1}{4}\pi \left(\frac{8R^3}{3\sqrt{3}}\right)$
 $= \frac{2\pi R^3}{\sqrt{3}} - \frac{2\pi R^3}{3\sqrt{3}}$
 $V_{maks} = \frac{6\pi R^3 - 2\pi R^3}{3\sqrt{3}} = \frac{4\pi R^3}{3\sqrt{3}}$

Figure 4. Example of Student's Encoding Error

Encoding error is the fifth error identified. Encoding error is the student's error in writing the answer correctly, unable to show the truth of the answer or not writing the conclusion of the answer [13,16]. In this study, 6.1% of students made mistakes. One of encoding errors the students did in this study is shown in Figure 4. In Figure 4, students initially experienced a reading error but did not fall into the classification of the error because the students did the example before making the mathematical model (tube in the ball). The error that the student did that was included as encoding error was if student did not write the conclusion of the answer and not being able to show the truth of the answer. Encoding error occurred because students were not accustomed to write conclusions after each work on math problems, let alone to check the truth of answers. Because the students thought that the most important thing was that they got answers from the given mathematical problem [17,18].

The last error identified is careless. Careless is also called error due to carelessness. Additionally, when students have read and understood the problem correctly but decided not to try to get answers is included in careless [13]. In this study, 19.4% of students did careless. Many students made this type of mistake because, students had read the problem well and had been correct in identifying what was known and asked of the problem given. But the student chose not to try to do the problem. Careless occurred because the student had never found the problem and worked on a problem like that. Students knew the concept that should be used was the concept of derived algebra but, students had

difficulty choosing what procedure should be done to solve the problem and in the end students chose not to do it [10,12]. The results showed five student errors in solving mathematical problems in derived algebra, i.e. comprehension error, transformation error, process skill error, encoding error and careless.

Among the errors that had been identified, the most frequent errors that students made were comprehension error, transformation error and process skill error. The results of this study are consistent with those of Clements and Erna and Budi that comprehension error, transformation error and process skill errors are more common than students' other errors of students [13,17]. The most influential factors in the three errors are the students' low-level reasoning and creativity abilities in solving mathematical problems and manipulating into algebra and unfamiliar use of correct problem-solving procedures [17]. The factor causing many students to misunderstand was that the students were not accustomed to identify and write the data being known and asked every time to solve a mathematical problem, since it was never examined by the teacher and not being treated as an error [18]. Similarly, encoding error, although a few students made mistakes, they were not used to write conclusions after each math problem, let alone to check the truth of answers very. Because the students think that the most important thing was that they got the answers from the given mathematical problem [17,18]. Although it looks trivial, but by familiarizing students to do math problems by initiating the identification of data being known and asked then ended with a conclusion and to solve the problem according to the steps of solving the problem correctly can indirectly decrease the mistakes the students make and familiarize students in working on the problem in a structured and clear.

4. Conclusion

Students need to know the mistakes they made when solving mathematical problems so that students can reflect to correct their mistakes and not repeat the same mistakes. The result of this research shows that there are five classifications of error that students did in solving the mathematical problem i.e. comprehension error, transformation error, process skill error, encoding error and careless. Teachers should give special attention to the mistakes made by students, such as telling students the mistakes when students do math problems at school and provide the right answer on the matter as well as teachers should be more selective in choosing the strategy, model or learning media that will be used in mathematics learning where it should emphasize students' understanding and problem solving ability to reduce student mistakes..

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